For the following questions, you may use the formulae for $\frac{d}{dx}\sinh x$ and/or $\frac{d}{dx}\cosh x$ without proving them. **SCORE: ______ / 6 PTS**

If you need to use the formula for the derivative of any other hyperbolic function, you must prove it.

[a] **Without using the logarithmic formula for $\cosh^{-1} x$**, prove the formula for $\frac{d}{dx}\cosh^{-1} x$.

$y = \cosh^{-1} x$

$\cosh y = x$

(1) $(\sinh y)\frac{dy}{dx} = 1$

(1) $\frac{dy}{dx} = \frac{1}{\sinh y}$

$= \frac{1}{\sqrt{x^2 - 1}}$ $\left(\frac{1}{2}\right)$

$\cosh^2 y - \sinh^2 y = 1$

$\left(\frac{1}{2}\right)$ $x^2 - \sinh^2 y = 1$

$\sinh^2 y = x^2 - 1$

BONUS (1) $\sinh y = \pm\sqrt{x^2 - 1}$

GRAPH OF $\cosh x$ IS 1-1 IF DOMAIN RESTRICTED TO $x \geq 0$

SO RANGE OF $\cosh^{-1} x$ IS $y \geq 0$

SO $\sinh y \geq 0$

[b] Prove the formula for $\frac{d}{dx}\coth x$.

$\frac{d}{dx}\frac{\cosh x}{\sinh x}$

$= \frac{(\sinh x)(\sinh x) - (\cosh x)(\cosh x)}{\sinh^2 x}$ (1)

(1) $= \frac{\sinh^2 x - \cosh^2 x}{\sinh^2 x}$ $\left(\frac{1}{2}\right)$ $= \frac{-1}{\sinh^2 x}$ $= -\operatorname{csch}^2 x$ $\left(\frac{1}{2}\right)$

Find $\lim_{x \to -\infty} \coth x$. **Do NOT use a graph. Give *BRIEF* algebraic or numerical reasoning.** **SCORE: ______ / 3 PTS**

$$= \lim_{x \to -\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$= \boxed{\lim_{x \to -\infty} \frac{e^{2x} + 1}{e^{2x} - 1}} \quad 1$$

$$= \boxed{\frac{0+1}{0-1}} \quad 1$$

$$= \boxed{-1} \quad 1$$

OR

$$= \boxed{\lim_{x \to -\infty} \frac{1 + e^{-2x}}{1 - e^{-2x}}} \quad 1 \qquad \frac{\infty}{-\infty}$$

$$= \boxed{\lim_{x \to -\infty} \frac{-2e^{-2x}}{2e^{-2x}}} \quad 1$$

$$= \boxed{-1} \quad 1$$

☆ GRADE AGAINST ONE VERSION ONLY

If $\tanh x = -\frac{1}{3}$, find $\sinh x$.

SCORE: ______ / 4 PTS

<u>You may use any hyperbolic identities from your textbook without proving them.</u>

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$1 - \frac{1}{9} = \operatorname{sech}^2 x$$

1 $\boxed{\operatorname{sech}^2 x = \frac{8}{9}}$

1 $\boxed{\operatorname{sech} x = \frac{2\sqrt{2}}{3}}$ SINCE $\operatorname{sech} x > 0$ FOR ALL x

1 $\boxed{\cosh x = \frac{3}{2\sqrt{2}}}$

$$\tanh x = \frac{\sinh x}{\cosh x} \longrightarrow \sinh x = \tanh x \cosh x$$

$$= -\frac{1}{3} \cdot \frac{3}{2\sqrt{2}} = \boxed{-\frac{1}{2\sqrt{2}} \text{ OR } -\frac{\sqrt{2}}{4}}$$

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EITHER ANSWER OK

Prove the logarithmic formula for $\sinh^{-1} x$ given in your textbook. **SCORE: ______ / 4 PTS**

NOTE: This is NOT a question about derivatives.

$$y = \sinh^{-1} x$$

$$\sinh y = x$$

$$\boxed{\frac{e^y - e^{-y}}{2} = x} \ \tfrac{1}{2}$$

LET $z = e^y$

$$\frac{z - \frac{1}{z}}{2} = x$$

$$z - \frac{1}{z} = 2x$$

$$1 \ \ z^2 - 1 = 2xz$$

$$\boxed{z^2 - 2xz - 1 = 0}$$

$$z = \boxed{\frac{2x \pm \sqrt{4x^2 + 4}}{2}} \ 1$$

$$z = \frac{2x \pm 2\sqrt{x^2+1}}{2}$$

$$e^y = \boxed{x \pm \sqrt{x^2+1}} \ \tfrac{1}{2} \quad x - \sqrt{x^2+1} < 0$$

$$\boxed{e^y = x + \sqrt{x^2+1}} \ \tfrac{1}{2}$$

$$y = \underline{\ln\left(x + \sqrt{x^2+1}\right)} = \sinh^{-1} x \ \ \tfrac{1}{2}$$

OK IF YOU USED e^y DIRECTLY INSTEAD OF z

Find $\frac{d}{dx} x^3 \sinh^{-1}(x^2)$. Simplify your final answer. **SCORE: ______ / 3 PTS**

<u>You may use the derivatives of any hyperbolic or inverse hyperbolic functions from your textbook without proving them.</u>

$$3x^2 \sinh^{-1} x^2 + x^3 \cdot \frac{1}{\sqrt{1+(x^2)^2}} 2x$$

$$= \boxed{3x^2 \sinh^{-1} x^2} + \frac{\boxed{2x^4}}{\boxed{\sqrt{1+x^4}}}$$

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